

1. (20%)

A linear time-invariant multivariable system is described by the following set of differential equations:

$$\frac{d^2 y_1(t)}{dt^2} + 2 \frac{dy_1(t)}{dt} + 3y_2(t) = r_1(t) + r_2(t)$$

$$\frac{d^2 y_2(t)}{dt^2} + 3 \frac{dy_2(t)}{dt} + y_1(t) - y_2(t) = r_2(t) + \frac{dr_1(t)}{dt}$$

Find the following transfer functions:

(a) $\frac{Y_1(s)}{R_1(s)}$ for $R_2(s) = 0$, (b) $\frac{Y_2(s)}{R_1(s)}$ for $R_2(s) = 0$,

(c) $\frac{Y_1(s)}{R_2(s)}$ for $R_1(s) = 0$, (d) $\frac{Y_2(s)}{R_2(s)}$ for $R_1(s) = 0$.

2. (10%)

Consider a system with following transfer function

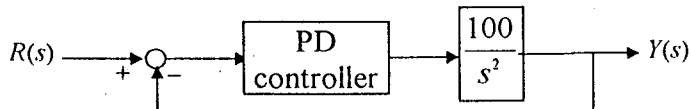
$$\frac{Y(s)}{U(s)} = \frac{2s^2 + 5s + 6}{s^3 + 5s^2 + 6s}$$

where y is the output and u is the input.

Find the dynamic equations in vector-matrix form such that the state equations are decoupled from each other.

3. (20%)

The block diagram of a PD control system is shown below



- Find K_P and K_D on which the damping is critical and the natural undamped frequency ω_n is 50 rad/sec.
- From part (a), with unit-step input, find the steady-state error.
- From part (a), with unit-step input, find the steady-state value of $y(t)$.
- Find K_P and K_D on which the parabolic-error constant K_a is 1000 sec^{-2} .

4. (20%)

Given the open-loop transfer function of a unity-feedback system

$$G(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)}; \text{ where } T_1 = 1, T_2 = 0.5.$$

- Draw the root locus for the system.
- Locate the roots for $K=0$, 0.125, and ∞ .
- If we add a zero at -5 , draw the new root locus and describe the effect on system stability.
- If we add a pole instead of a zero at -5 , draw the new root locus and describe the effect.

5. (10%)

The characteristic equation of a linear digital control system is

$$z^3 + z^2 + 1.5Kz - (K + 0.5) = 0$$

Determine the values of K for the system to be asymptotically stable.

6. (20%)

Consider the system with

$$G(s) = \frac{4(1 + \frac{s}{2})}{s(1 + 2s)(1 + \frac{s}{20} + (\frac{s}{8})^2)}$$

(a) Draw the Bode plot for the system.

(b) Draw the Bode plot for a lead compensator. Describe the effect if we add it to the system.

(c) Draw the Bode plot for a lag compensator. Describe the effect if we add it to the system.